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COMPUTATIONAL METHODS FOR LYAPUNOV FUNCTIONS

Lyapunov functions, introduced by Lyapunov more than 100 years ago, are to this day one of the most important tools in the stability analysis of dynamical systems. They are functions which decrease along solution trajectories of systems, and they can be used to show stability of an invariant set, such as an equilibrium, as well as to determine its basin of attraction. Lyapunov functions have been considered for a variety of dynamical systems, such as continuous-times, discrete-time, linear, non-linear, non-smooth, switched, etc. Lyapunov functions are used and studied in different communities, such as Mathematics, Informatics and Engineering, often using different notations and methods.

Since Lyapunov functions provide insight into the dynamics, it is an important question how to find them. A first answer is given by so-called converse theorems, which ensure the existence of a certain type of Lyapunov function given a certain type of stability. These theorems, however, are not constructive in nature as they usually use the solution trajectories to construct the Lyapunov function. This means, that they cannot be used directly to find an explicit Lyapunov function for most concrete examples. Therefore, computational methods have been derived to construct Lyapunov functions, using as diverse methods as optimization, Linear Matrix Inequalities, numerical solutions to Partial Differential Equations using collocation or other methods, graph theoretic methods, algebraic methods, and others.

This special issue brings together all these aspects: starting with two surveys on computational methods and converse theorems for Lyapunov functions and then continuing with six research articles which cover a wide range of current research for the construction of Lyapunov functions. The contributions also bring experts from different disciplines together in one volume.

The first contribution is *Review on Computational Methods for Lyapunov Functions* by the guest editors. It gives an overview over the different types of systems where Lyapunov functions are used before summarizing the various computational methods which have been employed for their construction. The second contribution is a survey by Christopher M. Kellett on *Classical Converse Theorems in Lyapunov's Second Method*. Converse theorems ensure the existence of Lyapunov functions for attractors. Their knowledge is therefore essential for anyone developing numerical procedures to compute Lyapunov functions.

The six remaining paper are research papers. James Anderson and Antonis Papachristodoulou study in *Advances in Computational Lyapunov Analysis using Sum-of-Squares Programming* recent developments in the field of computing polynomial Lyapunov functions using semidefinite optimization.

This paper is followed by the contribution *Polynomial Optimization with Applications to Stability Analysis and Control – Alternatives to Sum of Squares* by Reza Kamyar and Matthew M. Peet, where they study various alternatives to semidefinite optimization with application to the computation of polynomial Lyapunov functions.

In *Efficient Computation of Lyapunov Functions for Morse Decompositions* Arnaud Goullet, Shaun Harker, Konstantin Mischaikow, William D. Kalies, and Dinesh Kasti describe a graph-theoretic algorithm for the computation of a complete Lyapunov function for time-discrete systems.

In *Grid Refinement in the Construction of Lyapunov Functions using Radial Basis Functions* Najla Mohammed and Peter Giesl develop an adaptive scheme for grid refinement when solving a Zubov equation using collocation, whose solution is a Lyapunov function.

Computation of local ISS Lyapunov Functions with low Gains via Linear Programming by Huijuan Li, Robert Baier, Lars Grüne, Sigurdur F. Hafstein, and Fabian Wirth introduces a new algorithm which uses linear optimization to parameterize ISS Lyapunov functions securing input-to-state stability.

The final contribution, *Separable Lyapunov Functions for Monotone Systems: Constructions and Limitations* by Gunther Dirr, Hiroshi Ito, Anders Rantzer, and Björn S. Rüffer considers monotone systems, and the existence and construction of Lyapunov functions that can be separated into individual functions of scalar arguments.

We would like to thank all authors for their contributions, all anonymous reviewers for their work, and last but not least Peter Kloeden for giving us the opportunity for editing this issue. We hope that the contributions are of interest to a wide audience interested in Lyapunov functions in theory and practice and that this special issue helps bringing together the different communities working on stability in dynamical systems in the broadest sense.

Guest Editors:
Peter Giesl and Sigurdur Hafstein